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ANALYSIS OF CONTINUOUS BEAMS WITH JOINT SLIP.(U)
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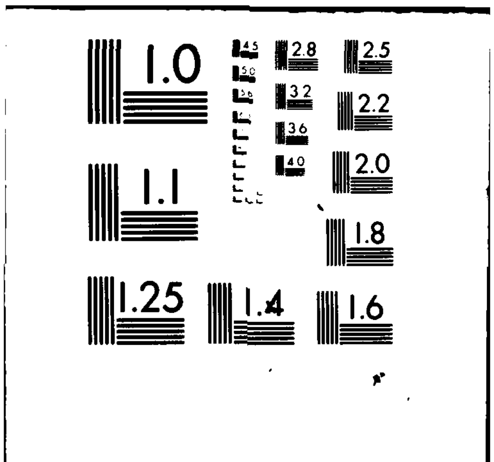

MICROCOPY RESOLUTION TEST CHART

United States Department of Agriculture

Forest Service

Forest Products Laboratory[1]

Research Note FPL-0244

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Analysis of Continuous Beams with Joint Slip

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Abstract

A computer analysis with user guidelines to analyze partially continuous multi-span beams is presented. Partial continuity is due to rotational slip which occurs at spliced joints at the supports of continuous beams such as floor joists. Beam properties, loads, and joint slip are input; internal forces, reactions, and deflections are output.

Introduction

Floor joist deflection and maximum design moment are decreased by utilizing continuous joists over two or more spans compared to simply supported joists. Splices are used to attain the lengths required for continuous members. The splices, whether nailed, bolted, glued, or truss plated, if located at an interior support, will have rotational slip occur when subjected to bending moment. Thus the beam continuity is disrupted with the joist acting somewhere between a simply supported and a fully continuous beam.

The finite element method of analysis, using discrete elements referred to as matrix structural analysis, is the state-of-the-art method used to analyze continuous structures. The usual procedure in matrix structural analysis to account for partial fixity is to model a short or fictitious member with a low stiffness value. This allows the input of some percentage of complete fixity but has the disadvantage of not being able to input the specific amount of joint slip since its relationship to the fictitious member stiffness is not known.

This study presents a computer analysis method and input user guidelines to determine internal forces, reactions and deflections of continuous beams with rotational slip at supports. Although developed specifically for two-span floor joist analysis and design, the method and computer program are applicable to any continuous beam structure.

Joist design is based on satisfying both stiffness and strength requirements. The stiffness criterion most commonly used is to limit maximum joist deflection in a floor system to span/360 when subjected to static live load of 40 pounds per square foot. The strength criterion limits bending stress to an allowable value based on species and grade of lumber. Design tables *(3)*[3] are available for floor joist design for either simple or two-span continuous beams. The continuous beam tables assume full moment capacity over the entire span (i.e., no splices or other loss of continuity).

The National Association of Homebuilders (NAHB) *(2)* and American Plywood Association (APA) *(1)* investigated two-span continuous beams with splices near the inflection points. The splices were designed to transmit shear force since moments near the inflection points are assumed small. Splice slip was not considered.

[1] In cooperation with U.S. Dep. of Housing and Urban Development.

[2] Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

[3] Italicized numbers in parenthesis refer to literature cited at the end of this report.

Splice slip is due to the behavior of the mechanical fastening system. Nails (and truss plates) slip due to bending of the nail (tooth) and crushing of the wood. Many studies have established nonlinear load-slip behavior for nailed joints in single shear. When bolts are used they are installed in oversized holes; thus a certain amount of movement occurs before loads are transmitted. Slip measurements related to design load levels are required for various splice configurations.

Splice slip affects both joist stiffness and strength. Slippage of a splice at an interior support will result in increased deflection throughout the span and in a decreased moment capacity at the support.

Matrix analysis *(4)* of continuous beams assumes supports either fixed or pinned. Intermediate end conditions are generally modeled by addition of a fictitious member with low stiffness or a rotational spring. Matrix analysis assumes each joint having discrete displacements corresponding to degrees of freedor. ; thus no discontinuity in displacement can occur. Loads at or between joints and support settlements can be included in the analysis.

Theory

The philosophy of the analysis is illustrated in figure 1. Deflections due to loads are found by matrix method assuming no joint slip. An experimentally determined slip, θ_s, is input and allocated, θ_r, θ_l, to the adjacent right and and left spans. Deflections caused by adjacent slips are calculated and superimposed on the deflections due to loads. Member end actions and support reactions are calculated from the member deflections.

The member stiffness matrix assumes the member being subjected to lateral loads which induce bending moments and shear forces. The effects of shear are neglected; thus two degrees of freedom at each node, vertical and rotational, are necessary. The degrees of freedom are numbered sequentially from left to right; thus the vertical translation of the left end is 1, the rotation 2, and those of the right end are 3 and 4, respectively, as shown in figure 2. Also shown is the member stiffness matrix with forces corresponding to unit displacements for each degree of freedom. The positive sign convention for the member forces and displacements are as shown, with vertical translation upward and rotation counterclockwise being positive.

The beam is modeled with nodes at points of support, changes in cross-section properties, and at any other point(s) where shear forces and bending moments, and/or vertical and rotational displacements are to be computed. Figure 3 shows node locations for an example two-span beam. It includes the support locations plus an arbitrary interior location, a distance x from the left support, where forces and displacements are to be calculated. Again, there are two degrees of freedom at each node; they are numbered sequentially starting at the left end of the beam. Member numbers (circled) are also sequential from the left end. Positive node forces and displacements are, as shown, upward and counterclockwise. The member stiffness matrices are superimposed, as shown symbolically, to form the structure stiffness matrix, [S].

The structure stiffness matrix is rearranged and partitioned related to the unknown displacements, D_d, and the known boundary displacements, D_s, representing support conditions:

$$[S] = \left[\begin{array}{c|c} S_{dd} & S_{ds} \\ \hline S_{sd} & S_{ss} \end{array}\right]$$

where the subscript s refers to support degrees of freedom (with known displacements of zero or support settlement value) and d refers to degrees of freedom with unknown displacements.

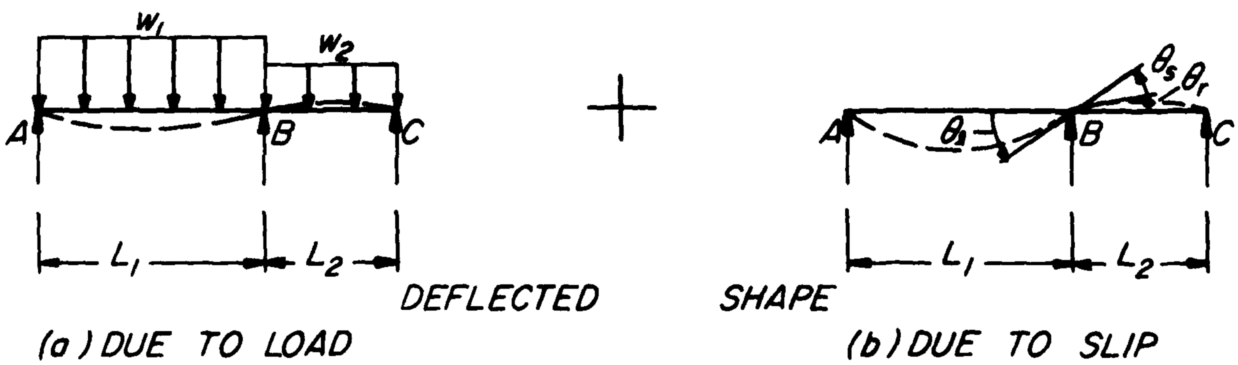
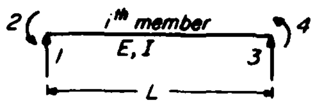


Figure 1.—Superposition of load and slip deflected shapes.

(M 149 237)



(a) Member

$$[S_m]_i = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & \frac{-12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & \frac{-6EI}{L^2} & \frac{2EI}{L} \\ \frac{-12EI}{L^3} & \frac{-6EI}{L^2} & \frac{12EI}{L^3} & \frac{-6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & \frac{-6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

(b) Member Stiffness Matrix

Figure 2.—Member degrees of freedom and stiffness matrix.

(M 149 238)

Node forces and/or support settlements (if any) are input with sign convention in figure 3b. Fixed end shears and moments corresponding to member loads are input with sign convention in figure 2a.

The unknown joint displacements, $\{D_d\}$, due to load, $\{A\}$, are found by:

$$\{D_d\} = [S_{dd}]^{-1}\{A\}$$
$$\{A\} = \{A_j\} - \{A_{fem}\} - [S_{ds}]\{D_s\}$$

where $\{A\}$ is the general load matrix consisting of the specified joint loads, $\{A_j\}$, fixed end reactions due to between-the-joint loads, $\{A_{fem}\}$, and the forces, $[S_{ds}]\{D_s\}$, due to support settlement, $\{D_s\}$.

The known joint slip is allocated to adjacent spans based on compatibility and equilibrium as indicated in figure 4 in which moments and rotations are shown in the positive direction. The effects of spans other than the adjacent spans are neglected. The sign convention for the slip rotation is that the slip angle is measured from the tangent to the elastic curve in the right span to the tangent of the elastic curve in the left span with counterclockwise rotation being positive.

From figure 4a, the compatibility equation is:

$$\theta_s = \theta_\ell - \theta_r \qquad (1)$$

where θ_s is the slip rotation, and θ_ℓ, θ_r are the rotational allocation of the slip to the left and right adjacent spans, respectively.

From figure 4b, the moment equilibrium equation at the interior support is:

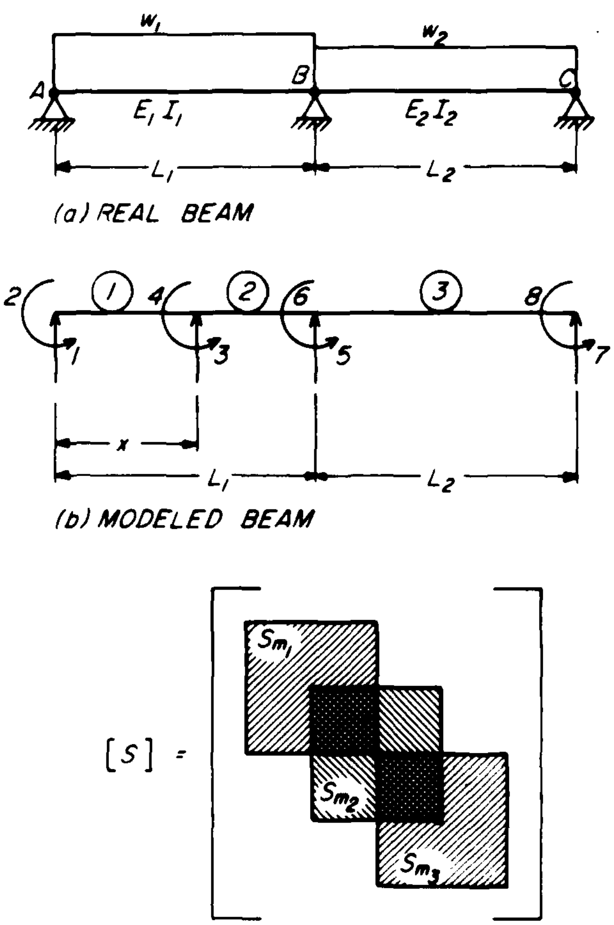
$$M_\ell = -M_r \qquad (2)$$

where M_ℓ, M_r are the moments corresponding to θ_ℓ and θ_r. From standard beam theory, the relationship between moment and rotation is:

$$M_\ell = \frac{3E_\ell I_\ell \theta_\ell}{L_\ell} \qquad (3a)$$

$$M_r = \frac{3E_r I_r \theta_r}{L_r} \qquad (3b)$$

where E is the modulus of elasticity, I the moment of inertia, and L the length of the left and right spans (subscripts ℓ, r), respectively. The relationship is applicable to members having a rotation at one end of the member.



(a) REAL BEAM

(b) MODELED BEAM

(c) STRUCTURE STIFFNESS MATRIX

Figure 3.—Structure degrees of freedom and stiffness matrix. *(M 149 239)*

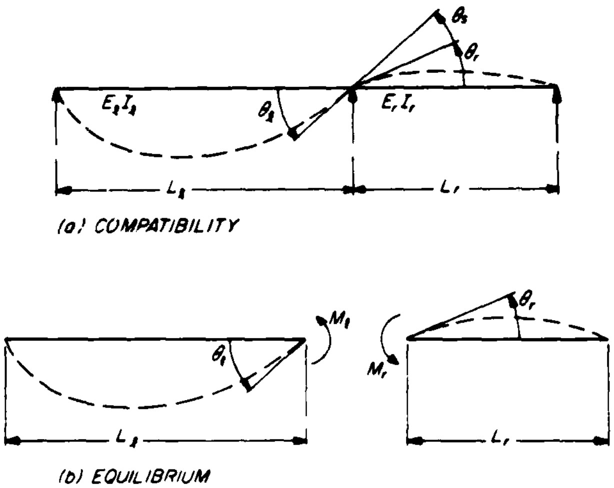


Figure 4.—Joint slip compatibility-equilibrium relations. *(M 149 240)*

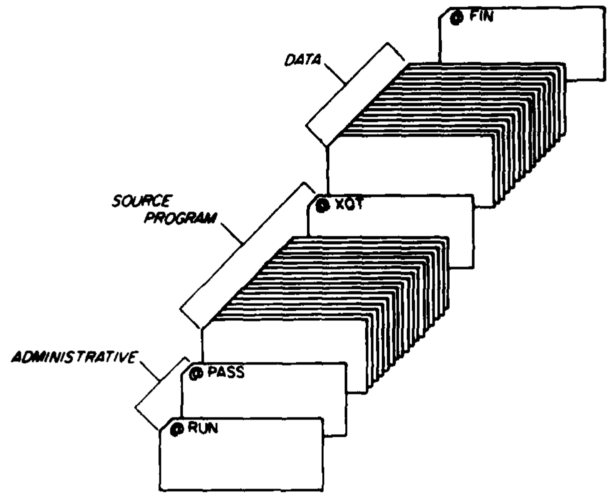


Figure 5.—Schematic representation of the makeup of computer deck. *(M 149 241)*

Substituting equations (3) into (2) yields a relationship between θ_ℓ and θ_r, and substituting this into equation (1) results in the following allocation of slip to adjacent spans (assuming $E_r = E_\ell$):

$$\theta_\ell = \theta_s \left[\frac{L_\ell I_r}{L_\ell I_r + L_r I_\ell}\right] \qquad (4a)$$

$$\theta_r = -\theta_s \left[\frac{L_r I_\ell}{L_\ell I_r + L_r I_\ell}\right] \qquad (4b)$$

To determine displacements due to a known slip, the continuous beam is separated into two structures at the support where the slip occurs. The slip allocations, θ_ℓ and θ_r, are applied as support displacements to each side of the separated structure. Deformations are then determined by matrix analysis.

The member end displacements due to slip are superimposed on the displacements due to loads. Member end displacements multiplied by the member stiffness matrix summed with the fixed end moments due to load result in member end forces. Adjacent member end forces combined with applied joint forces determine support reactions.

Procedure

The Fortran program using this theory is given in appendix A. The program is intended to be as complete as possible so that it can be easily modified for future research; it is not a production tool and no effort has been made to make it as efficient as possible. The units used must be compatible; the example in this paper uses input lengths in inches, forces in pounds, moments in inch-pounds, slip in radians, modulus of elasticity in pounds per square inch, and moment of inertia in inches^4. Output is in the same units.

The program is limited to one slip per member, and no slip at either the first or last support. Slips at adjacent joints are permissible provided each is indexed to a separate adjacent member. The program is arbitrarily limited to a maximum of 10 elements which is considered adequate for most 2- or 3-span floor joist systems (extra nodes may be included between supports). For larger problems, this limitation can be removed by modification of the dimension statement.

The degrees of freedom are numbered sequentially from the leftmost support with the vertical translation first and the rotation second. Sign conventions have been previously defined and are shown in the positive direction in figures 2, 3, and 4.

The Fortran program is stored on both tape and punched cards on the FPL/MACC system. Input data required are described in table 1. The command sequence for the FPL/MACC system to access the tape is given in table 2; that for the punched deck is shown in figure 5.

The following example illustrates both input data required and output generated.

Example

The two-span beam of figure 3a is assumed to have a splice at B which slips +0.00432 radian when loaded. Other values for this example are:

W_1 = 50 lb/ft = 4.16 lb/in. $I_1 = I_2$ = 20.8 in.^{4}

L_1 = 12 ft − 0 in. = 144 in. W_2 = 10 lb/ft = 0.833 lb/in.

$E_1 = E_2$ = 1,700,000 lb/in.^{2} L_2 = 9 ft − 0 in. = 108 in.

Shear and moment diagrams, and deformed shape for this partially continuous beam example are required.

Table 1.—Data cards

Number of cards required[1]	Information required[2]	Input data in columns numbered	Fortran format
1	a. NM = Total number of members	1 through 3*	I3
	b. NS = Total number of supports corresponding to degree of freedom numbers (i.e., NS = 2 for shear and moment at fixed support)	4 through 6*	I3
	c. NF = Total number of joint slips	7 through 9*	I3
	d. NA = Total number of degrees of freedom corresponding to joint loads (i.e., NA = 2 for joint with applied vertical load and moment)	10 through 12*	I3
NS	For each card (support):		
	a. Structure degree of freedom number	1 through 3*	I3
	b. Support settlement (in. or radians)	4 through 13	F10.6
NM	For each card (member):		
	a. Member number	1 through 3*	I3
	b. Modulus of elasticity (lb/in.^{2})	4 through 13	F10.0
	c. Moment of inertia (in.^{4})	14 through 21	F8.2
	d. Length (in.)	22 through 28	F7.2
	e. Left end fixed end shear due to member loads (lb)	29 through 37	F9.2
	f. Left end fixed end moment (in.-lb)	38 through 46	F9.2
	g. Right end fixed end shear (lb)	47 through 55	F9.2
	h. Right end fixed end moment (in.-lb)	56 through 64	F9.2
NA	For each card (joint load):		
	a. Structure degree of freedom number corresponding to joint load	1 through 3*	I3
	b. Joint load (lb or in.-lb)	4 through 13	F10.2
NF	a. Member number (may be either left or right span adjacent to slip)	1 through 3*	I3
	b. Member degree of freedom number (either 2 or 4 corresponding to member selected in "a")	4 through 6*	I3
	c. Structure degree of freedom number	7 through 9*	I3
	d. Slip (radians)	10 through 19	F10.6

[1] Cards must be sequenced in this order.
[2] Zero values may be entered by blanks.
* Values must be right-adjusted.

Table 2.—Tape command sequence

@RUN...
@PASS...
@CAT HUD*CONTINUBEAM.
@ASG, AX HUD*CONTINUBEAM.
@ASG, TH DIMEN*LUMBER.,U9H, 7639
@TGET DIMEN*LUMBER., HUD*CONTINUBEAM.
@XQT HUD*CONTINUBEAM.SLIP
Data as per table 1
@FIN

The beam is modeled with joints at points of support and at other arbitrary locations deemed necessary to define the shear and moment diagrams and the deformed shape. For illustrative purposes, only one arbitrary location 5 feet (60 in.) from the left support (x = 60 in.) is selected. Thus the beam is modeled with structure degrees of freedom and members numbered sequentially from the left as in figure 3b.

The input data corresponding to table 1 are:

Card 1

NM = 3
NS = 3 (*corresponds to structure degrees of freedom 1, 5, and 7*)
NF = 1 (*corresponds to structure degree of freedom 6*)
NA = 0 (*no joint loads, only member loads*)

Cards 2, 3, and 4, respectively
Structure degree of freedom = 1, 5, and 7, respectively
Support settlement = 0 (all cards)

Cards 5, 6, and 7, respectively
Member number = 1, 2, and 3, respectively
Modulus of elasticity = 1,700,000 lb/in.^{2} (all cards)
Moment of inertia = 20.8 in.^{4} (all cards)
Length = 60, 84, and 108 in., respectively
Left end fixed end shear for member 1 =

$$+\frac{W_1X}{2} = \frac{50 \times 5}{2} = +\ 125.0 \text{ lb}$$

Left end fixed end moment for member 1 =

$$+\frac{W_1X^2}{12} = \frac{50 \times 5^2 \times 12}{12} = +\ 1250.0 \text{ in.-lb}$$

Fixed end reactions for right end and for other members similarly found.
(Note, sign convention as per figure 2; thus right end fixed end moment is negative.)

No NA cards (NA = 0)

Card 8 (NF = 1)
Member number of left adjacent span = 2 (alternately, member 3, the right adjacent span could be selected)
Member degree of freedom (figure 2a) = 4 (alternately, member degree of freedom = 2 could be selected to correspond to member 3) Structure degree of freedom (figure 3b) = 6 Slip = + 0.00432 radian

The output forces and displacements are given in appendix B to illustrate the output format (which is referenced to the structure degree of freedom numbering). Joint displacements are not given directly since the slip creates a discontinuity; member end displacements to the left and right of the joint are given.

The results are plotted as case II in figure 6. Results for simply supported and fully continuous beams, found by standard structural analysis, are given as cases I and III for comparison. As expected, the behavior of the partially continuous beam is bounded by the simple and fully continuous cases. The displacement at x = 5 feet is 0.6318 and 0.4126 inch for the simply supported, and fully continuous beams, respectively. The partially continuous beam results in a deflection of 0.4738 inch. Use of a joint at B reduces the simply supported deflection by about 25 percent. The negative moment over the center support is reduced from the fully continuous value of 558 to 406 foot-pounds for the partially continuous beam; however,the corresponding positive moment at x = 5 feet is increased from 643 to 706 foot-pounds. This is still about 20 percent less than the simply supported positive moment. Figure 7 details the discontinuity of the deformed shape at support B.

Program Alteration

The program is arbitrarily limited to 10 elements (NM = 10); the corresponding number of joint degrees of freedom is 22 (2NM + 2). Dimensioned arrays have values of 10, 22, or 4 corresponding to number of elements, number of joint degrees of freedom or number of degrees of freedom per element. To increase the number of elements, change the dimension statements in the first six cards of the source program as follows:

a. Dimensions of value 10 are increased to the new number of elements, NM.
b. Dimensions of value 22 are increased to a value = 2NM + 2.
c. Dimensions of value 4 are unchanged.
d. Dimensions of the array called SCRACH are increased to 3 x (2NM + 2).

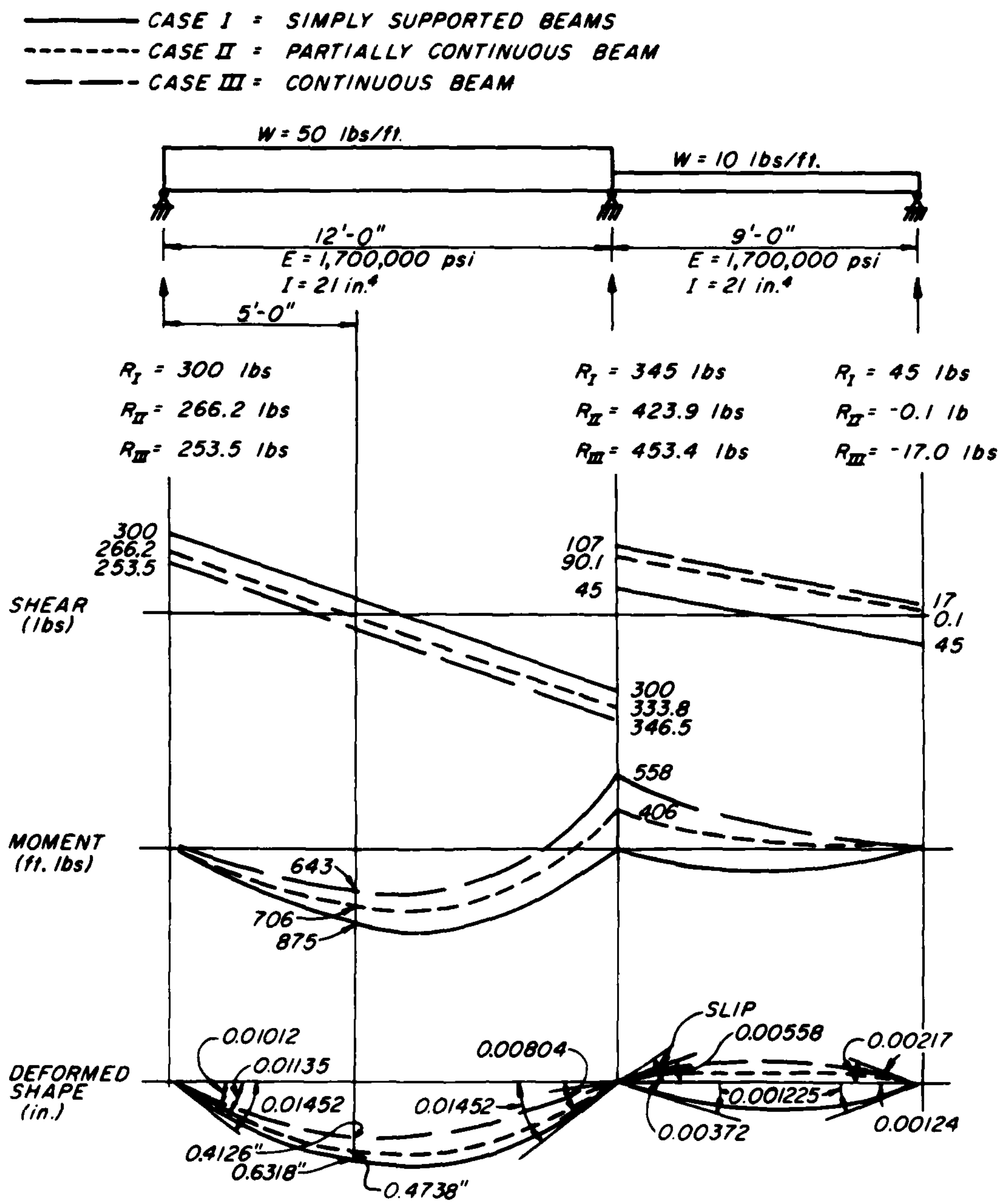


Figure 6.—Example-continuous beam with splice slip at support. (M 149 150)

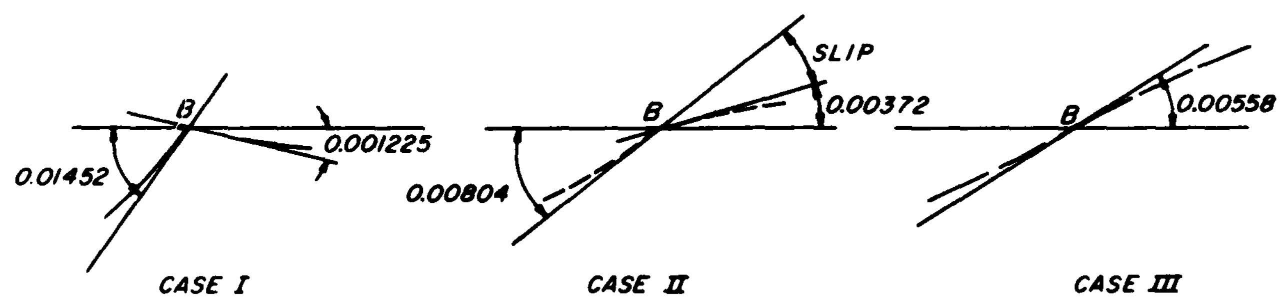


Figure 7.—Example-splice slip detail at support. (M 149 316)

Appendix A
Computer Source Program

```
C      MATRIX ANALYSIS OF CONTINUOUS BEAMS
C      MAXIMUM NUMBER OF SPANS = 10
       DIMENSION JNS(22),DS(22),JNSS(22),INS(22),AJ(22),SR(22),DDD(22), T
      1(22),B(10),V(10),W(10),X(10),SM(10,4,4),E(10),XI(10),XL(10),AM(10,
      24),AMF(10,4),COOR(22),XLS(10),S(22,22),A(22),JND(22),RR(22,22),R(2
      32,22),SDD(20,20),SSD(22,22),SDS(22,22),SSS(22,22),AS(22),AD(22),C(
      422),Y(22),SDDI(20,20),SCRACH(60,60),DD(22),D(22),SLIP(22),SLIP1(22
      5),SLIP2(22),DL(10,4),DR(10,4),AMM(10,4),AJJ(22)
C
C      INPUT DATA
       READ(5,1)NM,NS,NF,NA
       DO 5 I=1,NS
       READ(5,2)JNS(I),DS(I)
     5 JNSS(I)=JNS(I)
     1 FORMAT(4I3)
     2 FORMAT(I3,F10.6)
C
C      INITIALIZE CONDITIONS
       NIF=0.
       NSSJJ=0.
       NMM=NM
       NSS=NS
       DO 10 I=1,22
       INS(I)=0.
       AJJ(I)=0.
       AJ(I)=0.
       SR(I)=0.
    10 DDD(I)=0.
C
C      MEMBER STIFFNESS MATRIX
       DO 15 J=1,NM
       READ(5,3)M,E(M),XI(M),XL(M),(AM(M,I),I=1,4)
     3 FORMAT(I3,F10.0,F8.2,F7.2,4F9.2)
       B(M)=6.*E(M)*XI(M)/XL(M)**2.
       V(M)=2.*B(M)/XL(M)
       W(M)=4.*E(M)*XI(M)/XL(M)
       X(M)=W(M)/2.
       SM(M,1,1)=V(M)
       SM(M,2,2)=W(M)
       SM(M,3,3)=V(M)
       SM(M,4,4)=W(M)
       SM(M,1,3)=-V(M)
       SM(M,3,1)=-V(M)
       SM(M,2,4)=X(M)
       SM(M,4,2)=X(M)
       SM(M,1,2)=B(M)
       SM(M,1,4)=B(M)
       SM(M,2,1)=B(M)
       SM(M,4,1)=B(M)
       SM(M,3,2)=-B(M)
       SM(M,3,4)=-B(M)
       SM(M,2,3)=-B(M)
    15 SM(M,4,3)=-B(M)
       DO 16 J=1,NMM
       DO 16 L=1,4
    16 AMF(J,L)=AM(J,L)
C
C      STRUCTURE SPAN LENGTHS
       NJ=2*NM+2
```

```
      NOJ=NJ
      COOR(1)=0.
      DO 20 I=3,NJ,2
      K=(I-1)/2
   20 COOR(I)=COOR(I-2)+XL(K)
      NSL=NS-1
      I=1
      DO 35 J=3,NJ,2
      DO 30 K=2,NS
      IF(J.EQ.JNS(K))GO TO 25
      GO TO 30
   25 JJ=JNS(K-1)
      XLS(I)=COOR(J)-COOR(JJ)
      I=I+1
      GO TO 35
   30 CONTINUE
   35 CONTINUE
C
C     STRUCTURE JOINT LOADS
      IF(NA)50,50,40
   40 DO 45 I=1,NA
      READ(5,4)NAM,AJ(NAM)
    4 FORMAT(I3,F10.2)
      AJJ(NAM)=AJ(NAM)
   45 CONTINUE
   50 CONTINUE
C
  909 CONTINUE
C
C     STRUCTURE STIFFNESS AND LOAD MATRIX
      DO 55 I=1,22
      A(I)=0.
      DO 55 J=1,22
   55 S(I,J)=0.
      DO 75 I=1,NM
      M=I
      M1=2*M-1
      M2-2*M
      M3=2*M+1
      M4=2*M+2
      M=M+NSSJJ/2
      S(M1,M3)=SM(M,1,3)
      S(M1,M4)=SM(M,1,4)
      S(M2,M3)=SM(M,2,3)
      S(M2,M4)=SM(M,2,4)
      S(M3,M1)=SM(M,3,1)
      S(M3,M2)=SM(M,3,2)
      S(M4,M1)=SM(M,4,1)
      S(M4,M2)=SM(M,4,2)
      S(M1,M1)=S(M1,M1)+SM(M,1,1)
      S(M1,M2)=S(M1,M2)+SM(M,1,2)
      S(M2,M1)=S(M2,M1)+SM(M,2,1)
      S(M2,M2)=S(M2,M2)+SM(M,2,2)
      IF(I-NM)60,65,70
   60 S(M3,M3)=S(M3,M3)+SM(M,3,3)
      S(M3,M4)=S(M3,M4)+SM(M,3,4)
      S(M4,M3)=S(M4,M3)+SM(M,4,3)
      S(M4,M4)=S(M4,M4)+SM(M,4,4)
      GO TO 70
   65 S(M3,M3)=SM(M,3,3)
      S(M3,M4)=SM(M,3,4)
      S(M4,M3)=SM(M,4,3)
```

```
      S(M4,M4)=SM(M,4,4)
   70 CONTINUE
      A(M1)=A(M1)-AM(M,1)+AJ(M1)
      A(M2)=A(M2)-AM(M,2)+AJ(M2)
      A(M3)=A(M3)-AM(M,3)+AJ(M3)
      A(M4)=A(M4)-AM(M,4)+AJ(M4)
   75 CONTINUE
C
C     REARRANGE STIFFNESS MATRIX FOR JOINT RESTRAINT
C     DETERMINE JND
      L=1
      DO 95 I=1,NJ
      DO 90 K=1,NS
      IF(I-JNS(K))80,95,80
   80 CONTINUE
      IF(K-NS)90,85,90
   85 JND(L)=I
      L=L+1
   90 CONTINUE
   95 CONTINUE
C     MOVE VERTICAL COLUMNS
      ND=NJ-NS
      DO 100 I=1,NJ
      DO 100 J=1,ND
      KK=JND(J)
  100 PR(I,J)=S(I,KK)
      DO 105 I=1,NJ
      DO 105 J=1,NS
      KK=JNS(J)
  105 RR(I,J+ND)=S(I,KK)
C     MOVE HORIZONTAL ROWS
      DO 110 J=1,NJ
      DO 110 I=1,ND
      KK=JND(I)
  110 R(I,J)=RR(KK,J)
      DO 115 J=1,NJ
      DO 115 I=1,NS
      KK=JNS(I)
  115 R(I+ND,J)=RR(KK,J)
C     TO SUBDIVIDE MATRIX
      DO 120 I=1,ND
      DO 120 J=1,ND
  120 SDD(I,J)=R(I,J)
      DO 125 I=1,NS
      DO 125 J=1,ND
  125 SSD(I,J)=R(I+ND,J)
      DO 130 I=1,ND
      DO 130 J=1,NS
  130 SDS(I,J)=R(I,J+ND)
      DO 135 I=1,NS
      DO 135 J=1,NS
  135 SSS(I,J)=R(I+ND,J+ND)
C     REARRANGE LOAD MATRIX
      DO 140 I=1,ND
      K=JND(I)
  140 AD(I)=A(K)
      DO 145 I=1,NS
      K=JNS(I)
  145 AS(I)=A(K)
C
C     TO FIND JOINT DISPLACEMENTS
      DO 150 I=1,ND
```

```
      C(I)=0.
      DO 150 K=1.NS
  150 C(I)=C(I)+SDS(I.K)*DS(K)
      DO 155 I=1.ND
  155 Y(I)=AD(I)-C(I)
      CALL MTINV2(SDD.SDDI.ND.20.20.'GEN'.0.$610.SCRACH)
      DO 160 I=1.ND
      DD(I)=0.
      DO 160 K=1.ND
  160 DD(I)=DD(I)+SDDI(I.K)*Y(K)
      DO 162 I=1.ND
      J=JND(I)+NSSJJ
      DDD(J)=DD(I)+DDD(J)
  162 D(J)=DD(I)
      DO 165 I=1.NS
      K=JNS(I)+NSSJJ
      DDD(K)=DS(I)+DDD(K)
  165 D(K)=DS(I)
C
C     IF SLIP AT SUPPORT OCCURS
      IF(NF)340.340.170
  170 CONTINUE
      DO 175 I=1.NMM
      DO 175 J=1.4
      DO 175 K=1.22
      AM(I.J)=0.
      AJ(K)=0.
  175 DS(K)=0.
      IF(NIF-2*NF)180.340.340
  180 NIF=NIF+1
      KNIF=NIF/NF
      GO TO (190.275).KNIF
  190 CONTINUE
      READ(5.6)M.KKK.NFM.SLIP(NFM)
    6 FORMAT(3I3.F10.6)
      INS(M)=NFM
      NSJ=2*((NFM+1)/2)-1
      L=0
      DO 195 I=1.NSS
      IF(JNSS(I).EQ.NSJ) GO TO 200
      IF(JNSS(I).EQ.NFM) GO TO 200
  195 L=L+1
  200 GO TO(210.205.210.205).KKK
  205 SLIP1(NFM)=SLIP(NFM)*XLS(L)*XI(M+1)/(XI(M+1)*XLS(L)+XI(M)*XLS(L+1)
     1)
      SLIP2(NFM)=SLIP1(NFM)-SLIP(NFM)
  210 CONTINUE
C
C     FIND NUMBER OF JOINTS AND MEMBERS IN DIVIDED STRUCTURE
      NLJ=2*((NFM+1)/2)
      NLM=(NLJ-2)/2
      NRM=NMM-NLM
      NRJ=2*NRM+2
C     DETERMINE NUMBER OF SUPPORTS IN LEFT SPANS
      L=0
      DO 220 I=1.NLJ
      DO 220 J=1.NSS
      IF(I-JNSS(J))220.215.220
  215 L=L+1
  220 CONTINUE
      DO 225 I=1.NSS
      IF(JNSS(I).EQ.NSJ)GO TO 230
      IF(JNSS(I).EQ.NFM)GO TO 230
```

```
  225 CONTINUE
      GO TO 235
  230 L=L-1
  235 CONTINUE
      GO TO (250.245.250.245).KKK
  245 NLS=L+2
      JNS(NLS-1)=2*((NFM-1)/2)+1
      JNS(NLS)=NFM
      DS(NLS)=SLIP1(NFM)
  250 CONTINUE
      NM=NLM
      NJ=NLJ
      NS=NLS
      GO TO 999
  275 CONTINUE
C     DETERMINE NUMBER OF SUPPORTS IN RIGHT SPANS
      KNFM=2*((NFM-1)/2)+1
      NSSJJ=NSJ-1
      L=0
      DO 285 I=KNFM.NOJ
      DO 285 J=1.NSS
      IF(I-JNSS(J))285.280.285
  280 L=L+1
  285 CONTINUE
      DO 290 I=1.NSS
      IF(JNSS(I).EQ.NSJ)GO TO 295
      IF(JNSS(I).EQ.NFM)GO TO 295
  290 CONTINUE
      GO TO 300
  295 L=L-1
  300 GO TO (325.310.325.310).KKK
  310 NRS=L+2
      JNS(2)=NFM-NSSJJ
      JNS(1)=NFM-1-NSSJJ
      DS(2)=SLIP2(NFM)
      LM=2
      DO 320 I=1.NSS
      IF(JNSS(I).GT.NFM)GO TO 315
      GO TO 320
  315 LM=LM+1
      JNS(LM)=JNSS(I)-NSSJJ
  320 CONTINUE
  325 CONTINUE
      NM=NRM
      NJ=NRJ
      NS=NRS
      GO TO 999
  340 CONTINUE
C
C     TO SUPERIMPOSE DEFLECTIONS AND FIND MEMBER END FORCES
      DO 345 M=1.NMM
      M1=2*M-1
      M2=2*M
      M3=2*M+1
      M4=2*M+2
      DR(M.1)=DDD(M1)
      DR(M.2)=DDD(M2)
      DR(M.3)=DDD(M3)
  345 DR(M.4)=DDD(M4)
      DO 395 M=1.NMM
      IF(NF)350.395.350
  350 K=INS(M)
      M1=2*M-1
```

```
      M2=2*M
      M3=2*M+1
      M4=2*M+2
      IF(K.GE.M1.AND.K.LE.M4)GO TO 355
      GO TO 395
  355 KK=K-2*M+2
      GO TO (395.360.395.370).KK
  360 DL(M.2)=DL(M.2)-SLIP1(K)
      IF(M-1)365.395.365
  365 DR(M-1.4)=DR(M-1.4)-SLIP2(K)
      GO TO 395
  370 DR(M.4)=DR(M.4)-SLIP2(K)
      IF(M-NMM)375.395.375
  375 DL(M+1.2)=DL(M+1.2)-SLIP1(K)
  395 CONTINUE
      DO 400 M=1.NMM
      DO 400 J=1.4
  400 AMM(M.J)=SM(M.J.1)*DL(M.1)+SM(M.J.2)*DL(M.2)+SM(M.J.3)*DR(M.3)+SM(
     1M.J.4)*DR(M.4)+AMF(M.J)
C
C     TO DETERMINE SUPPORT REACTIONS
      T(1)=AMM(1.1)-AJJ(1)
      T(2)=AMM(1.2)-AJJ(2)
      IF(NMM.EQ.1)GO TO 410
      NMML=NMM-1
      NMMT=2*NMM
      DO 405 M=1.NMML
      DO 405 I=3.NMMT.2
      T(I)=AMM(M.3)+AMM(M+1.1)-AJJ(I)
  405 T(I+1)=AMM(M.4)+AMM(M+1.2)-AJJ(I+1)
  410 T(2*NMM+1)=AMM(NMM.3)-AJJ(2*NMM+1)
      T(2*NMM+2)=AMM(NMM.4)-AJJ(2*NMM+2)
      DO 415 I=1.NSS
      K=JNSS(I)
  415 SR(I)=T(K)
C
C     PRINT OUTPUT
      WRITE(6.500)
  500 FORMAT('1'.32X.'MATRIX ANALYSIS OF CONTINUOUS BEAM'.////)
      WRITE(6.505)
  505 FORMAT(1X.'MEMBER'.5X.'LENGTH'.9X.'E'.9X.'I'.4X.'LEFT SHEAR'.3X.'L
     1EFT MOMENT'.3X.'RIGHT SHEAR'.3X.'RIGHT MOMENT'.//)
      DO 515 M=1.NMM
      WRITE(6.510)M.XL(M).E(M).XI(M).(AMM(M.J).J=1.4)
  510 FORMAT(1X.I3.8X.F5.0.6X.F9.0.1X.F7.1.4X.F7.1.5X.F8.1.8X.F7.1.6X.F8
     1.1./)
  515 CONTINUE
      WRITE(6.520)
  520 FORMAT(/.1X.'DEGREE OF FREEDOM'.21X.'VERTICAL DISPLACEMENT'.21X.'R
     1OTATION'./)
      WRITE(6.525)
  525 FORMAT(42X.'LEFT'.6X.'RIGHT'.22X.'LEFT'.6X.'RIGHT')
      WRITE(6.530)
  530 FORMAT(43X.'OF'.9X.'OF'.24X.'OF'.9X.'OF')
      WRITE(6.535)
  535 FORMAT(41X.'JOINT'.6X.'JOINT'.21X.'JOINT'.6X.'JOINT'.//)
      I=1
      WRITE(6.540)I.DL(1.1)
  540 FORMAT(7X.I3.38X.F10.5)
      NJJ=NMM-1
      DO 550 M=1.NJJ
      I=I+2
      WRITE(6.545)I.DR(M.3).DL(M+1.1)
```

```
545 FORMAT(7X.I3.26X.F10.5.2X.F10.5)
550 CONTINUE
    I=I+2
    WRITE(6.555)I.DR(NMM.3)
555 FORMAT(7X.I3.26X.F10.5)
    I=2
    WRITE(6.560)I.DL(1.2)
560 FORMAT(7X.I3.75X.F10.5)
    DO 570 M=1.NJJ
    I=I+2
    WRITE(6.565)I.DR(M.4).DL(M+1.2)
565 FORMAT(7X.I3.63X.F10.5.2X.F10.5)
570 CONTINUE
    I=I+2
    WRITE(6.575)I.DR(NMM.4)
575 FORMAT(7X.I3.63X.F10.5)
    WRITE(6.580)
580 FORMAT(//.1X.'DEGREE OF FREEDOM',7X,'REACTION',7X,'MOMENT',//)
    DO 605 I=1.NSS
    K=JNSS(I)
    KK=(K+1)/2
    KJ=K/2
    IF(KJ-KK)595.595,585
585 CONTINUE
    WRITE(6.590)K.SP(I)
590 FORMAT(7X.I3.13X.F9.2)
    GO TO 605
595 CONTINUE
    WRITE(6.600)K.SP(I)
600 FORMAT(7X.I3.27X.F9.2)
605 CONTINUE
610 CONTINUE
    STOP
    END
```

*** STATEMENT NUMBERS ***

1	11	*15	
2	13	*16	
3	32	*33	
4	81	*82	
5	12	*14	
6	217	*218	
10	23	*28	
15	31	*53	
16	54	55	*56
20	62	*64	
25	69	*71	
30	66	70	*75
35	67	74	*76
40	79	*80	
45	80	*84	
50	79	*85	
55	90	92	*93
60	113	*114	
65	113	*119	
70	113	118	*123
75	94	*128	
80	135	*136	
85	137	*138	
90	134	137	*140

95	133	135	*141					
100	144	145	*147					
105	148	149	*151					
110	153	154	*156					
115	157	158	*160					
120	162	163	*164					
125	165	166	*167					
130	168	169	*170					
135	171	172	*173					
140	175	*177						
145	178	*180						
150	183	185	*186					
155	187	*188						
160	190	192	*193					
162	194	*197						
165	199	*201						
170	204	*205						
175	206	207	208	*211				
180	212	*213						
190	215	*216						
195	222	*225						
200	223	224	*226					
205	226	*227						
210	226	*230						
215	241	*242						
220	239	240	241	*243				
225	244	*247						
230	245	246	*249					
235	248	*250						
245	251	*252						
250	351	*256						
275	215	*261						
280	268	*269						
285	266	267	268	*270				
290	271	*274						
295	272	273	*276					
300	275	*277						
310	277	*278						
315	284	*286						
320	283	285	*288					
325	277	*289						
340	204	212	*294					
345	297	*305						
350	307	*308						
355	313	*315						
360	316	*317						
365	318	*319						
370	316	*321						
375	322	*323						
395	306	307	314	316	318	320	322	*324
400	325	326	*327					
405	335	337	*339					
410	333	*340						
415	342	*344						
500	347	*348						
505	349	*350						
510	353	*354						
515	355	*356						
520	357	*358						
525	360	*361						
530	362	*363						
535	364	*365						
540	367	*368						

```
545         372  *373
550         370  *374
555         376  *377
560         379  *380
565         383  *384
570         381  *385
575         387  *388
580         389  *390
585         395  *396
590         397  *398
595         395  *400
600         401  *402
605         391   399   *403
610         189  *404
999         *87   260   293
```

*** VARIABLES ***

```
A              3    *91  *124  *125  *126  *127   177   180
AD             3   *177   188
AI             3    *26   *81    83   124   125   126   127  *210
AII            3    *25   *83   331   332   338   339   340   341
AN             3    *32    56   124   125   126   127  *209
ANF            3    *56   327
ANN            3   *327   331   332   338   339   340   341   353
AS             3   *180
B              3    *34    35    46    47    48    49    50    51    52    53
C              3   *184  *186   188
COOP           3    *61   *64    72
D              3   *197  *201
DD             3   *191  *193   196   197
DDD            3    *28  *196  *200   302   303   304   305
DL             3   *302  *303  *317  *323   327   367   372   379   383
DR             3   *304  *305  *319  *321   327   372   376   383   387
DS             3    *13   186   200   201  *211  *255  *281
E              3    *32    34    36   353
I            *12    *13    14   *23    24    25    26    27    28   *32   *62    63
              64    *66    72   *73   *80   *90    91    93   *94    95   113  *133
             135    138  *144   147  *148   151  *154   155   156  *158   159   160
            *162    164  *165   167  *168   170  *171   173  *175   176   177  *178
             179    180  *183   184   186  *187   188  *190   191   193  *194   195
             196    197  *198   199   200   201  *206   209  *222   223   224  *239
             241   *244   245   246  *266   268  *271   272   273  *283   284   287
            *337    338   339  *342   343   344  *366   367  *371   372  *375   376
            *378    379  *382   383  *386   387  *391   392   397   401
INS            3    *24  *219   308
J            *31    *54    56   *67    69    72   *92    93  *145   146   147  *149
             150    151  *153   156  *157   160  *163   164  *166   167  *169   170
            *172    173  *195   196   197  *207   209  *240   241  *267   268  *326
             327    353
JJ           *71     72
JND            3   *138   146   155   176   195
JNS            3    *13    14    69    71   135   150   159   179   199  *253  *254
            *279   *280  *287
JNSS           3    *14   223   224   241   245   246   268   272   273   284   287
             343    392
K            *63     64   *68    69    71  *134   135   137  *176   177  *179   180
            *185    186  *192   193  *199   200   201  *208   210   211  *308   313
             315    317   319   321   323  *343   344  *392   393   394   397   401
KJ          *394    395
```

[illegible]	[illegible]	147	*150	151	*155	156	*159	160	*315	316	*393	395
[illegible]	*217	226	251	277								
[illegible]	[illegible]	266										
[illegible]	*214	215										
[illegible]	[illegible]	56	*132	138	*139	*221	*225	227	*238	*242	*249	252
	[illegible]	*263	*276	278								
[illegible]	[illegible]	*286	287									
[illegible]	[illegible]	34	35	36	37	38	39	40	41	42	43	44
	[illegible]	46	47	48	49	50	51	52	53	*95	96	97
	[illegible]	99	*100	101	102	103	104	105	106	107	108	109
	110	111	112	114	115	116	117	119	120	121	122	124
	[illegible]	126	127	*217	219	227	*297	298	299	300	301	302
	[illegible]	304	305	*306	308	309	310	311	312	315	317	318
	319	321	322	323	*325	327	*336	338	339	*352	353	*370
	372	*381	383									
M1	*96	101	102	105	107	109	110	111	124	*298	302	*309
	313											
M2	*97	103	104	106	108	110	111	112	125	*299	303	*310
M3	*98	101	103	105	106	114	115	116	119	120	121	126
	*300	304	*311									
M4	*99	102	104	107	108	115	116	117	120	121	122	127
	*301	305	*312	313								
MTINV2	[illegible]											
[illegible]	[illegible]	79	80									
[illegible]	*81	83										
[illegible]	*137	145	151	154	160	162	163	166	167	168	170	173
	175	183	187	189	190	192	194					
NF	[illegible]	204	212	214	307							
[illegible]	*217	219	220	224	227	229	233	246	253	254	255	263
	273	279	280	281	284							
[illegible]	[illegible]	212	*213	214								
[illegible]	*59	60	62	67	133	143	144	148	153	157	*258	*291
[illegible]	*309	370	381									
[illegible]	*233	234	239	258								
[illegible]	*234	235	257									
[illegible]	*252	253	254	255	259							
[illegible]	[illegible]	21	31	59	94	113	*257	*290				
[illegible]	[illegible]	54	206	235	297	306	322	325	333	334	335	340
	[illegible]	352	369	376	387							
[illegible]	*334	336										
[illegible]	*335	337										
[illegible]	[illegible]	266										
[illegible]	[illegible]	291										
[illegible]	[illegible]	276	290									
[illegible]	[illegible]	92										
[illegible]	[illegible]	12	22	65	68	134	137	143	149	158	165	169
	[illegible]	172	178	195	198	*259	*292					
[illegible]	[illegible]	[illegible]	[illegible]	264	272							
[illegible]	[illegible]	[illegible]	210	244	267	271	283	342	391			
[illegible]	[illegible]	100	195	199	*264	279	280	287				
F	[illegible]	*156	*160	164	167	170	173					
FF	[illegible]	*147	*151	156	160							
S	[illegible]	*99	*101	*102	*103	*104	*105	*106	*107	*108	*109	*110
	[illegible]	*112	*114	*115	*116	*117	*119	*120	*121	*122	147	151
SCRATCH	[illegible]	184										
[illegible]	[illegible]	*164	[illegible]									
[illegible]	[illegible]	189	193									
[illegible]	[illegible]	*170	186									
[illegible]	[illegible]	*217	227	229								
[illegible]	[illegible]	[illegible]	229	255	317	323						
[illegible]	[illegible]	*229	281	319	321							

[illegible]	[illegible]	+38	+39	+40	+41	+42	+43	+44	+45	+46	+47	+49
	+48	+50	+51	+52	+53	101	102	103	104	105	106	107
	[illegible]	+109	110	111	112	114	115	116	117	119	120	121
		[illegible]										
[illegible]	[illegible]	+[illegible]	+311	397	401							
[illegible]		+167										
[illegible]		+177										
[illegible]	[illegible]	+331	+332	+338	+339	*340	*341	344				
[illegible]	[illegible]	+35	38	40	42	43						
[illegible]	[illegible]	+36	37	39	41							
	[illegible]	+37	44	45								
I	[illegible]	+32	34	36	227	353						
I	[illegible]	+32	34	35	36	64	353					
[illegible]	[illegible]	+72	227									
[illegible]	[illegible]	+188	193									

Appendix B

Example Output

MATRIX ANALYSIS OF CONTINUOUS BEAM

MEMBER	LENGTH	E	I	LEFT SHEAR	LEFT MOMENT	RIGHT SHEAR	RIGHT MOMENT
1	60.	1700000.	20.8	266.2	-.0	-16.2	8469.3
2	84.	1700000.	20.8	16.2	-8469.3	333.8	-4873.6
3	108.	1700000.	20.8	90.1	4873.6	-.1	.0

DEGREE OF FREEDOM	VERTICAL DISPLACEMENT		ROTATION	
	LEFT OF JOINT	RIGHT OF JOINT	LEFT OF JOINT	RIGHT OF JOINT
1		.00000		
3	-.47382	-.47382		
5	.00000	.00000		
7	.00000			
2				-.01135
4			-.00205	-.00205
6			.00804	.00572
8			-.00124	

DEGREE OF FREEDOM	REACTION	MOMENT
1	266.16	
5	423.97	
7	-.13	

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4. Willems, N. and W. M. Lucas, Jr.
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C02-11-2

U.S. Forest Products Laboratory

Analysis of Continuous Beams with Joint Slip, by Lawrence Soltis, Madison, Wis., FPL 1981.
20 p. (USDA For. Serv. Res. Note FPL 0244).

A computer analysis method and input user guideline to determine internal forces, reactions and deflections of continuous beams with rotational slip at supports is presented. The method and computer program are applicable to any continuous beam structure, although developed specifically for two-span floor joist analysis and design.

Keywords: Joist design, rotational slip, two-span floor joist, deflections, continuous beams.

2.0-20-7/81

☆ U. S. GOVERNMENT PRINTING OFFICE: 1981/750-027/13